Exam Code: 225702 Paper Code: 2225

Programme: Master of Science (Mathematics) Semester-II

Course Title: Measure Theory

Course Code: MMSL – 2331

Time Allowed: 3 Hours Max. Marks: 70

Note: Attempt five questions in all, selecting atleast one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks

Section –A

Q-1 (a) Define Lebesgue outer measure or Outer measure. State and Prove its any four properties

(b) Prove that the collection M of all measurable sets is a σ -algebra of sets in P(R) (power set of R)

[7,7]

Q-2 (a)Define continuity in measure and State and Prove Borel Cantelli lemma

(b) Prove that there exist a non measurable set in the interval [0,1)

[7,7]

Section-B

Q-3 a) Define Characteristic function on a set E and prove that it is a measurable function iff E is measurable

b) Let f be a measurable function defined on E. Then there exist a sequence $.f_n$ of continuous functions on R s.t.$f_n \to f$ a.e. on E

[7,7]

Q-4 a) Prove that sum , product and composition of two measurable set is measurable

b) State and prove Lusin's Theorem

[7,7]

Section –C

Q-5(a) Prove that a bounded function f defined on a measurable set E with finite measure is Lebesgue integral iff f is measurable

(b) Prove that every Riemnann Integrable function is Lebesgue Integrable but Converse is not true.

[7,7]

Q-6 a)Define Integral of non negative measurable function. Also State and prove its linearity , and additivity over domain of integration

b) State and prove Monotone convergence theorem with its application

[7,7]

Section- D

Q-7 a) State and Prove Generalised Lebesgue Dominated Convergence Theorem

b) Let f be a non negative function which is integrable over a set E . Then for given $\in > 0, \exists \delta > 0$ *such that for every set* $A \subset E$ *with* $m(A) < \delta$, we have $\int f < \in$ over the Set A

[7,7]

Q-8 a) Show that the function f is not Lebesgue integrable on [0,1] where f(x) $=\frac{1}{x}$ for $0 < x \leq 1$ *and* $f(x) = 0$ *for* $x = 0$

b) State and Prove Vitali Convergence Theorem

[7,7]

Exam Code: 225702 **Paper Code: 2226**

Programme: Master of Science (Mathematics) Semester-II

Course Title: Linear Algebra

Course Code: MMSL-2332

Time Allowed: 3 Hours **Max. Marks: 70**

Note: Candidates are required to attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 14 marks.

SECTION-A

1) (a) State and Prove rank Nullity theorem.

(b) Let $T: V \to W$ be a linear transformation where V and W are vector spaces over the field F. Prove that $N_T = \{0\}$ iff T carries each linearly independent subset of V onto a linearly independent subset of W. (7,7)

2) (a) Find a basis and dimension of the subspace W of R^4 generated by the following vectors:

(1,2,3,5), (2,3,5,8), (3,4,7,11), (1,1,2,3). Also extend these to a basis of R^4.

(b) Let V be a vector space over the field F. If T is an invertible linear operator on V, then prove that $TT^{-1} = I = T^{-1}T$ (7,7)

SECTION-B

3) (a) Define the rank of a matrix. Find the rank of the following matrix by reducing it to echelon form:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 3 & 5 \end{pmatrix}$$

(b) Solve the following system of linear equations using matrices:

$$2x + 3y - z = 5$$
$$4x + y + 2z = 6$$
$$-2x + 5y + z = -3$$ (7,7)

4) (a) Let $T: R^3 \rightarrow R^2$ be a linear transformation defined by

$$T(x, y, z) = (2x + y - z, 3x - 2y + 4z).$$

Find the matrix of T relative to ordered basis $B_1 = \{(1,1,1), (1,1,0), (1,0,0)\}$ *and* $B_2 = \{(1,3), (1,4)\}$ of R^3 *and* R^2 respectively.

(b) Let T be a linear operator on R^2 defined by T(x,y)=(4x-2y, 2x+y)

(i) Find the matrix T relative to the basis B=$\{(1,1); (-1,0)\}$

(ii) Also verify that $[T; B][v; B] = [T(v); B]$ for any vector vϵR^2.

(7,7)

SECTION-C

5) (a) If λ is an eigen value of a linear operator T, then λ^n is an eigen value of T^n for every positive integer n.

(b) Check whether the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \text{ is diagonalizable.}$$

6) (a) If W_1 *and* W_2 are subspace of a finite dimensional vector space V and $V = W_1 \oplus W_2$ then $W^*{}_1 \simeq W^0{}_2$ and $W^*{}_2 \simeq W^0{}_1$.

(b) Prove that every matrix satisfies its characteristic equation. (7,7)

SECTION-D

7) (a) Prove that R^2 is an inner product space with an inner product defined by

$$< x, y > = x_1y_1 - 2x_1y_2 - 2x_2y_1 + 5x_2y_2$$

$$where\ x =< x_1, x_2 > \&\ y =< y_1, y_2 >$$

(b) Let T be a unitary operator on inner product space V. Let W be subspace invariant under T. Then $W^\perp$ is also invariant under T. (7,7)

8) Prove that every finite dimensional inner product space has an orthogonal basis. (14)

Exam Code: 225702

Paper Code: 2227

Programme: Master of Science (Mathematics) Semester-II

Course Title: Algebra-II

Course Code: MMSL-2333

Time Allowed: 3 Hours

Max. Marks: 70

Note: Candidates are required to attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 14 marks.

SECTION-A

1. (a) Prove that every finite integral domain is a field.

   (b) Show that $p\boldsymbol{Z}$ is a maximal ideal iff p is a prime number.

2. (a) State and prove Fundamental theorem of Ring Homomorphism.

   (b) Prove that an ideal M of a commutative ring with unity is a maximal ideal if and only if R/M is a field.

SECTION-B

3. (a) Show that 3 is an irreducible element but not prime in $Z[\sqrt{-5}]$.

   (b) Prove that every irreducible element in principal ideal domain is prime element.

4. (a) Prove that in a Unique Factorization Domain R an element is prime if and only if it is irreducible.

   (b) State and prove Gauss theorem.

SECTION-C

5. (a) Prove that every field must have a prime subfield.

   (b) Show that $\boldsymbol{Q}(\sqrt{5}, i)$ is a simple extension of Q.

6. (a) Find the degree of the splitting field of $X^4 - 5X^2 + 6$ over Q.

   (b) If L is an algebraic extension of K and K is an algebraic extension of F then L is an algebraic extension of F.

SECTION-D

7. (a) Prove that every finite separable extension is simple extension.

(b) Prove that every algebraic extension of a perfect field is perfect.

8. (a) Prove that it is impossible by straight edge and compass alone to trisect 60^0.

(b) Show that the regular n-gon is constructible if and only if $\phi(n)$ is a power of 2.

Exam Code: 225702 **Paper Code: 2228**
(20)

Programme: Master of Science (Mathematics) Semester-II

Course Title: Mechanics-II

Course Code: MMSL-2334

Time Allowed: 3 Hours **Max Marks: 70**

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question can be attempted from any Section. Each question carries 14 marks.

Section-A

Q1 A uniform heavy solid hemisphere of radius 'a' is held at rest with its base vertical and its curved surface in contact with a horizontal plane. If the hemisphere is released when the plane is rough enough to prevent slipping, show that the angle θ that the base makes with the horizontal at time 't' is such that

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{15\, g\, Cos\,\theta}{a(28-15\, Cos\,\theta\,)}$$ 14

Q2(a) A uniform rod is placed on a horizontal table with two thirds of its length hanging over the edge of the table.If the rod is at right angles to the edge and is released. Show that it will begin to slip when the rod has turned through an angle of $\tan^{-1}(\frac{\mu}{2})$ where μ is the coefficient of friction between the rod and the table. 7

(b) The cross-section of a right cylinder of mass M is bounded by a parabolic arc and the latus rectum of length 2l of the parabola. The surface of the cylinder is smooth and it is placed with its rectangular face on a horizontal plane. A particle of mass m is placed at the mid-point of the highest generator of the cylinder and slightly disturbed. Using the principles of conservation of energy and linear momentum, find the speed of the cylinder and the speed of the particle relative to the cylinder when the particle still in contact with the surface has descended a vertical distance x. 7

Section -B

Q3(a) Derive Euler's Dynamical equations for the motion of a rigid body about a fixed point. 10

(b) Prove that the first component of the angular momentum of a rigid body rotating with one point fixed is

$Aw_1 - Fw_2 - E\,w_3$ in the usual notation. 4

Q4 (a) Define Polhode and Herpolhode and find their equations. 7

(b) Find the kinetic energy of a rigid body rotating about a fixed point with velocity $\vec{w}$. 7

Section- C

Q5 Write a short note on any two :

(1) Linear Harmonic Oscillator
(2) Dumb Bell
(3) Atwood's Machine 14

Q6. A uniform heavy rod AB of mass m and length 12a is suspended from a fixed point O by a light string OC of length 5a attached to a point C of the rod so that AC = 4a. The system makes small oscillations about its time t. Prove that the kinetic energy of the system is $\frac{1}{2} ma^2 (25\dot{q}_1^2 + 20\, \dot{q}_1\dot{q}_2 + 16\, \dot{q}_2^2)$ and that the normal periods of oscillations are $\frac{2\pi}{p_1}$ and $\frac{2\pi}{p_2}$ where $3ap_1^2 = g$ and $10\, a\, p_2^2 = g$. 14

Section-D

Q7(a) State and prove Brachistochrone problem. 7

(b) Find the extremals of the functional

$$\int_0^{\pi/2} \{ 2xy + (\tfrac{dx}{dt})^2 + (\tfrac{dy}{dt})^2 \}\, dt$$ 7

Q8(a) Find the extremals $\int_0^1 y'^2\, dx$ given that y(0)=1 , y(1) = 6 and

$\int_0^1 y\, dx = 3$ 7

(b) Solve the Boundary Value Problem

$$y'' + (1+ x^2)\, y + 1 = 0$$

such that y (1) = 0 , y(-1) = 0 by Rayleigh -Ritz method. 7

Exam Code: 225702 Paper Code: 2229

Programme: Master of Science (Mathematics) Semester – II

Course Title: Number Theory

Course Code: MMSL-2335

Time Allowed: - 3 Hours Max. Marks: - 70

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 14 marks.

Section A

Q1. (a) Find all roots to the congruence $x^3 + 2x - 3 \equiv 0\ mod(9)$.

(b) Using Chinese Remainder Theorem, solve the congruence $17x \equiv 9\ (mod\ 276)$. [7x2=14]

Q2. (a) Justify whether that every prime number has primitive root or not.

(b) Find all primitive roots modulo 5^2. [7x2=14]

Section B

Q3. (a) Solve $3x^5 \equiv 1\ (mod\ 23)$, using the index table, with 5 as the smallest primitive root of 23.

(b) Is $x^2 \equiv 73\ (mod\ 173)$ solvable? Justify your answer. [7x2=14]

Q4. (a) State and Prove Generalized Quadratic Reciprocity Law for distinct co prime natural numbers m and n.

(b) If p is an odd prime, than prove that $x^2 \equiv 2\ (mod\ p)$ has a solution iff $p \equiv 1\ or\ 7\ (mod\ 8)$. [7x2=14]

Section C

Q5. (a) State and prove Mobius Inversion formula.

(b) If a positive integer n has r distinct prime factors $p_1, p_2, \ldots, p_r$, then justify that

$$\sum_{d|n} \mu(d)\sigma(d) = (-1)^r \, p_1 p_2 \cdots p_r.$$

[7x2=14]

Q6. (a) Prove or disapprove: An odd prime p is sum of two squares if and only if $p \equiv 1 \ (mod\ 4)$.

(b) Find all solutions of $x^2 + y^2 = z^2$, such that $0 < z < 30$.

[7x2=14]

Section D

Q7 (a) If $\frac{a}{b}, \frac{a'}{b'}$ are consecutive fractions in F_n, say with $\frac{a}{b}$ to the left of $\frac{a'}{b'}$ then justify that

$$a'b - ab' = 1.$$

(b) Find the successor of $\frac{7}{19}$ in F_{39}, the set of Farey fractions of order 39.

[7x2=14]

Q8 (a) Find the simple continued fraction for $\sqrt{41}$.

(b) Find the fundamental solution of equation $x^2 - 41y^2 = 1$.

[7x2=14]

Exam Code: 225704 Paper Code: 4212
(20)

Programme: Master of Science (Mathematics) Semester-IV

Course Title: Functional Analysis-II
Course Code: MMSL-4331

Time Allowed: 3 Hours Max Marks: 80

Note: Attempt five questions in all, selecting one question from each section. Fifth question can be attempted from any Section. Each question carries 16 marks.

Section -A

Q-1(a) *If* $< x_n >$ *and* $< y_n >$ are sequences in a same normed space x , such that$< x_n >$ weakly converges to x *and* $< y_n >$ weakly converges to y then show that $< x_n + y_n >$ weakly converges to x+y as well as $\alpha < x_n >$ *converges weakly to x ,where α is any scalar*

(b) *If* $< x_n >$ *weakly converges to* x_0 in a Normed linear space X : then show that $x_0 \in \bar{Y}$, where y = Span x_n (8,8)

2(a) Give an example of an operator which is (i) Self adjoint as well unitary (ii) normal but not unitary

(b) Prove that if T is an operator on H then the following conditions are equivalent to one another: (i) T*T = I (ii) (Tx,Ty) =(x,y) ∀ x and y (iii) ||Tx||=||x|| ∀ x (8,8)

Section- B

3(a) Let X be a Complex Banach Space and T∈ $B(X)$.Then, the Spectrum σ (T) is a closed subset of complex plane.

(b) If T be a unitary operator on a Hilbert space H, then the characteristic values of T are unimodular. (8,8)

4. State and Prove Spectral Mapping theorem for polynomials (16)

Section -C

5.(a) Let X be complex Banach space and $T \in B(X)$b then prove that (i) The resolvent set ρ is an open set

(ii)The Spectrum $\sigma(T)$ is closed subset of complex plane

(b) Let $T_1: N \to N'$ and $T_2: N \to N'$, are compact operator where N and N'are normed space. Show that $T_1 + T_2$ are compact linear operator (8,8)

6(a) Let $\{T_n\}$ be a sequence of compact linear operator from normed space N into Banach space N'. If $\{T_n\}$ is uniformaly operator convergent thyen prove that limit T of operator is compact .

(b) Show that $T: l^2 \to l^2$ defined by $y = T(x) = y_j$ where $y_j = \frac{x_j}{j}$, j= 1,2,3,......., is compact (8,8)

Section -D

7(a) Let A be a complex Banach algebra with identity . if $x \in A$ *such that* $||x|| < 1$. Then (e-x) is invertible and $(e-x)^{-1} = e + \sum_{n=1}^{\infty} x^n$. Also show that $||(e-x)^{-1} - (e-x) \leq \frac{||x||^2}{1-||x||}$

(b) If $\{x_n\}$ *and* $\{y_n\}$ are Cauchy sequence in Normed algebra A. Show that $[x_n\ y_n\}$ is Cauchy Sequence in A (8,8)

8(a) Prove that the boundary of set G^c is the subset of Z.

(b) Let 'a' be a complex Division algebra .then show that 'a'is isomorphic to set C of all Complex number (8,8)

Exam Code: 225704 **Paper Code: 4213**
(20)

Programme: Master of Science (Mathematics) Semester-IV

Course Title: Topology-II

Course Code: MMSL-4332

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any Section. Each question carries 16 marks.

Section- A

1. (a) Define Completely Regular Topological space. Also prove completely regularity is a hereditary property. (8)
(b) Prove that every metric space is completely normal and hence T_5 space (8).

2. Define Tychonoff space with the help of an example. Let $(X_\alpha, \tau_\alpha)_{\alpha\epsilon\Lambda}$ be topological spaces And (X, τ) be their product space then (X, τ) is Tychonoff if each (X_α, τ_α) is Tychono (16)
iff

Section- B

3. (a) If every cover of X by base elements has a finite subcover, then prove that X is compact. (8)
(b) Prove that every compact Hausdorff space is Regular and hence T_3 space. (8)

4. (a) State and Prove Tychonoff theorem. (8)
(b) Define Locally compact space. Prove that every compact topological space is locally compact but converse is not true. (8)

Section- C

5. State and prove Uryshon's Metrization theorem. (16)
6. (a) Prove that $R \times R$ with product topology is metrizable. (8)
(b) Prove that a topological space (X, τ) is Tychonoff iff it is homeomorphic to some subspace of Tychonoff cube. (8)

Section- D

7. (a) A point x in a topological space (X, τ) is a cluster point of a net S in X iff some subnet of S converges to x. (8)
(b) A topological space (X, τ) is Hausdorff iff limit point of each net in it is unique. (8)

8. (a) State and prove characterization of compactness in terms of filters. (8)
(b) Define Base and subbase of a filter and prove that a topological space is Hausdorff space iff every convergent filter on X has a unique limit. (8)

Exam Code: 225704 Paper Code: 4214
(20)

Programme: Master of Science (Mathematics)
Semester-IV

Course Title: Number Theory

Course Code: MMSL-4333 (Opt-VII)

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any Section. Each question carries 16 marks.

Section A

Q1. (a) Solve the congruence $x^5 - 3x^2 + 2 \equiv 0 \ (mod\ 7)$.

(b) Using Chinese Remainder Theorem solve the system of given congruences:
$x \equiv 2 \ (mod\ 3),\ \ x \equiv 3 \ (mod\ 5),\ \ x \equiv 2 \ (mod\ 7)$. [8x2=16]

Q2. (a) Show that any prime number p has $\emptyset\ (p-1)$ primitive roots.

(b) Find a primitive root modulo 5. Hence find all primitive roots for 50. [8x2=16]

Section B

Q3. (a) Given that 2 is smallest primitive root mod 13, construct the index table (mod 13) and hence solve $7^x \equiv 5 \ (mod\ 13)$.

(b) State and Prove Gauss Quadratic Reciprocity Law for prime numbers. [8x2=16]

Q4. (a) Evaluate the Legendre symbol $\left(\frac{-23}{59}\right)$.

(b) Evaluate $\left(\frac{231}{1105}\right)$. [8x2=16]

Section C

Q5. State and prove Lagrange's four square theorem. [16]

Q6. (a) Verify Mobius inversion formula for $n = 24$.

(b) If n is any even integer, check whether $\sum_{d|n} \mu(d)\phi(d) = 0$, where the symbols have the usual meaning. [8x2=16]

Section D

Q7 (a) Find the following and preceding terms of $\frac{2}{7}$ in F_{50}, the set of Farey fractions of order 50.

(b) Given any irrational number, ξ, prove that there exist infinitely many rational numbers $\frac{h}{k}$ such that $\left|\xi - \frac{h}{k}\right| < \frac{1}{\sqrt{5}k^2}$. [8x2=16]

Q8 (a) Determine the unique irrational number represented by [3, 6, 1, 4, 1, 4,......].

(b) Find the fundamental solution of $x^2 - 7y^2 = 1$. [8x2=16]

Exam Code: 225704 (20) | Paper Code: 4215

Programme: Master of Science (Mathematics)
Semester-IV

Course Title: Statistics-II

Course Code: MMSL-4334 (Opt-VIII)

Time Allowed: 3 Hours | Max Marks: 80

Note: Attempt five questions in all, selecting one from each section. Fifth question can be attempted from any Section. Each question carries equal 16 marks. Students are allowed to use statistical tables and a simple non programmable and non storage calculator.

Section -A

Q-1 a)Prove that $\frac{ns^2}{\sigma^2}$ is distributed like chi-square with n-1 d.f where s^2 and σ^2 are the variance of sample and population respectively.

Q-2 a) Derive the relation between t,F and Chi-square distribution.

b) Let $x_1, x_2, x_3 \ldots x_n$ be a random sample with common p.d.f

$$f(x)=\begin{cases} 1, 0 < x < 1 \\ 0, otherwise \end{cases} \quad H$$

Find the p.d.f , mean and variance of $x_{(1)}$

Section -B

Q -3 a)Explain the methods of moments of point estimation with an example

b)Obtain M.L.E.of θ in f(x,θ)=(1+θ)x^θ, $0 < x < 1$ based on an independent sample of size n. Examine whether this estimate is sufficient for θ

Q-4(a)If x1,x2,x3, …… ,xn is a random sample from a distribution having probability density function $f(x,\theta) = \begin{cases} \theta x^{\theta-1}, \theta < x \le 1 \\ 0, elsewhere \end{cases}$

Find a Best critical region for testing $H_0: \theta = 1$ *against* $H_1: \theta = 2$

(b) let p be the proportion of smokers in a certain city . it is purposed to test the hypothesis Ho: p=1/2 against p=3/4. If we reject Ho when 60 persons or more are found to be smokers in a sample of 100 persons , compute the level of significance and the power of the test.

Section C

Q-5 (a) In one sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level, given that critical point for (7,9) d.f. is 3.29.

b) Explain F-test as a test of equality of two population Variances

Q-6 (a) The number of scooter accidents per month in a certain town was as follows:

12,8,20,2,14,10,15,6,9,4

Use Chi-Square test to determine if these frequencies are in agreement with the belief that accident conditions were the same during 10 months period.

6(b) Find likelihood ratio test for the mean of a normal distribution

Section D

Q-7 The following data provides the yield of wheat on 12 plots divided into three classes on the basis of three varieties of fertilizers

X1	X2	X3
6	3	3
7	5	3
5	7	5
9	6	2

Is there any significant difference between average yields of wheat under three varieties of fertilizers x1, x2, x3

Q-8 Define ANOVA . What are its assumptions? Discuss model and analysis of variance of one way classified data

Exam Code: 225704
(20)

Paper Code: 4216

Programme: Master of Science (Mathematics)
Semester-IV

Course Title: Operations Research-II

Course Code: MMSL-4335 (Opt-IX)

Time Allowed: 3 Hours

Max Marks: 80

Note: Candidates are required to attempt five question by selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks. The students can use only non-programmable & non-Storage type calculator.

SECTION-A

1. (a) Discuss the queueing system and its various elements. (8 Marks)

(b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following:
(i) the mean queue size (line length), and
(ii) the probability that the queue size exceeds 10. (8 Marks)

2. (a) Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines, one of which is reserved for shunting purposes), calculate the probability that the yard is empty and find the average queue length. (8 Marks)

(b) Explain the generalized model: Birth-Death process. (8 Marks)

SECTION-B

3. (a) What are the different costs associated with inventories. (8 Marks)

(b) A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been assigned to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year? (8 Marks)

4. (a) Explain the concept of deterministic inventory problem with shortages. (8 Marks)

(b) Find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	Unit Cost (Rs.)
$0 \leq Q_1 < 800$	1.00
$800 \leq Q_2$	0.98

The yearly demand for the product is 1600 units, the cost of placing an order is Rs. 5, the cost of storage is 10% per year. (8 Marks)

SECTION-C

5. (a) A firm is considering replacement of a machine, whose cost price is Rs. 12200 and the scrap value, Rs. 200. The running (maintenance and operating) costs in rupees are found from experience to be as follows:

Year	1	2	3	4	5	6	7	8
Running cost	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced? (8 Marks)

(b) In a machine shop, a particular cutting tool costs Rs. 6 to replace. If a tool breaks on the job, the production disruption and associate costs amount to Rs. 30. The past life of a tool is given as follows:

Job No.	1	2	3	4	5	6	7
Proportion of broken tools on job	.01	.03	.09	.13	.25	.55	.95

After how many jobs should the shop replace a tool before it breaks down? (8 Marks)

6. (a) A research team is planned to raise to a strength of 50 chemists and then to remain at that level. The number of recruits depends on their length of service which is as follows:

Year	1	2	3	4	5	6	7	8	9	10
%age left at year ends	5	36	56	63	68	73	79	87	97	100

What is the recruitment per year necessary to maintain the required strength? (8 Marks)

(b) Prove that the renewal rate of a machine is asymptotically reciprocal of the mean life of the machine. (8 Marks)

SECTION-D

7. (a) Discuss the following:

(i) Methodology of simulation (ii) Simulation Models. (8 Marks)

(b) At a telephone booth, suppose that the customers arrive with an average time of 1.2 time units between one arrival and the next. Service times are assumed to be 2.8 time units. Simulate the system for 12 time units by assuming that the system starts at $t = 0$. What is the average waiting time per customer? (8 Marks)

8. (a) Write a short note on Monte-Carlo simulation. (8 Marks)

(b) A confectioner sells confectionery items. Past data of demand per week (in hundred kilograms) with frequency is given below:

Demand/week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Consider the following sequence of random numbers:

35, 52, 90, 13, 23, 73, 34, 57, 35, 83, 94, 56, 67, 66, 60

Using the sequence, generate the demand for the next 10 weeks. (8 Marks)