Exam Code: 225703
(20)

Paper Code: 3213

Programme: Master of Science (Mathematics)
Semester-III

Course Title: Functional Analysis

Course Code: MMSL-3331

Time Allowed: 3 Hours

Max Marks: 70

Note: Attempt five questions in total by selecting at least one question from each section A-D. The fifth question may be selected from any Section. Each question carries 14 marks.

Section-A

1. a) Prove that the space $l_p^{\,n}$ is a Banach space. 7

   b) State and prove Holder's inequality. 7

2. a) Prove that all norms on a finite dimensional norm linear space are equivalent. 7

b) Let N and M be normed linear space over the same field of scalers and T is a linear transformation from N to M. Then T is bounded iff T takes bounded sets in N to bounded sets in M. 7

Section-B

3. a) Prove that a finite dimensional norm linear space is complete. 7

b) If x and y are different elements of a norm linear space N then there exists f in N^* Such that $f(x) \neq f(y)$ 7

4. a) Show that there exists natural imbedding of N into N**. Where N is a norm linear space.

7

b) If M is a closed linear subspace of a norm linear space N and x_0 is a vector not in M. Then there exists a functional f_0 in N* s.t. f_0 (M)=0 but $f_0(x_0) \neq 0$.

7

Section-C

5. State and prove closed graph theorem.

14

6. Let T be an operator on a norm linear space N, prove that its conjugate T* is an operator on N* and ϕ : $\beta(N) \rightarrow \beta(N^*)$ defined by ϕ (T) =T* is an isometric isomorphism of β(N) into β (N*), Which reverses product and preserves identity.

14

Section-D

7. a) State and prove projection theorem.

9

b) State and prove Bessel's inequality for finite orthonormal sets. 5

8. a) State and prove Riesz-Representation theorem for continuous linear functional on Hilbert space.

9

b) If M is a linear subspace of Hilbert space, show that

M is closed iff M = $M^{\perp\perp}$. 5

Exam Code: 225703 Paper Code: 3214

Programme: Master of Science (Mathematics)

Semester – III

Course Title: Partial Differential Equations and Integral Equations

Course Code: MMSL-3332

Time Allowed: 3 Hours Max. Marks: 70

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

Section A

1 (a) Find the general solution of the differential equation
$$x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z.$$ 7

(b) Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1, z=1.$ 7

2 (a) Find the complete integral of the equations
$$p+q=pq.$$ 7

(b) Solve by Jacobi's method $z^2=pqxy.$ 7

Section B

3. (a) Find the surface satisfying $t=6x^3y$ and containing the two lines $y=0=z, y=1=z.$ 7

(b) Solve by Monge's Method $r=a^2t.$ 7

4. (a) Solve by the method of separation of variables
$$\frac{\partial^2 z}{\partial x^2}-\frac{2\partial z}{\partial x}+\frac{\partial z}{\partial y}=0.$$ 7

(b) Obtain the general solution of the one-dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$. 7

Section C

5 (a) Show that function F(x) =$(1 + x^2)^{\frac{-3}{2}}$ is a solution of the Volterra integral equation

$$F(x) = \frac{1}{1+x^2} - \int_0^x \frac{\eta}{1+x^2} F(\eta)d\eta.$$ 7

(b) Solve the following linear integral equations:

$$F(x) = x + \int_0^x (\eta - x)F(\eta)d\eta.$$ 7

6 (a) Solve the Volterra's integral equation of second kind, by using method of successive approximation:

$F(x) = 1 + \int_0^x F(\eta)d\eta$, with $F_o(x) = 0$. 7

(b) Solve by method of resolvent Kernel

$$F(x) = 1 + \lambda \int_0^1 (1 - 3x\eta)F(\eta)d\eta.$$ 7

Section D

7(a) Find the iterated Kernels for the following Kernel

$K(x,\eta) = e^x cos\eta$: $a = 0, b = \pi$. 7

(b) Solve the integral equation

$$F(x) = 1 + \lambda \int_0^1 (1 - 3x\eta)F(\eta)d\eta$$

For what values of λ the solution does not exist. 7

8 (a) Using Fredholm determinants, find the resolvent kernel of the following Kernels:

$K(x,\eta) = xe^\eta$; $a = 0, b = 1$. 7

(b) Show that the integral equation

$g(s) = f(s) + \frac{1}{\pi}\int_0^{2\pi} \sin(s+t)\, g(t)dt$ possesses infinite number of solution when $f(s) = 1$. 7

Exam Code: 225703 **Paper Code: 3215**

Programme: Master of Science (Mathematics)

Semester – III

Course Title: Operational Research-I

Course Code: MMSL-3333

Time Allowed: 3 Hours **Max. Marks: 70**

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks. *Only Non-Programmable and Non-Storage Type Calculators are allowed.*

Section-A

1. Solve by using Big-M method the following linear programming problem:

$$Max\ z = -2x_1 - x_2\ , subject\ to\ 3x_1 + x_2 = 3,$$
$$4x_1 + 3x_2 \geq 6,\ \ x_1 + 2x_2 \leq 4 \quad and\ x_1 \geq 0, x_2 \geq 0$$

2. Use Two-Phase simplex method to solve the problem :

$$Minimize\ \ z = x_1 - 2x_2 - 3x_3$$
$$subject\ to\ the\ constraints\ -2x_1 + x_2 + 3x_3 = 2,$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$and\ \ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Section-B

3. State and Prove Complementary Slackness Theorem.
4. Use Branch-and-Bond technique to solve the following integer programming problem:

$$Max\ z = 7x_1 + 9x_2\ ,$$
$$subject\ to\ -x_1 + 3x_2 \leq 6, \qquad 7x_1 + x_2 \leq 35\ ,$$
$$0 \leq x_1,\ x_2 \leq 7\ \ and\ x_1, x_2\ \ are\ integers$$

Section-C

5. Determine an initial basic feasible solution using
   i) Vogel's method
   ii) North-West corner method by considering the following transportation problem

		A_1	B_1	C_1	D_1	E_1	Supply
Origin	A	2	11	10	3	7	4
	B	1	4	7	2	1	8
	C	3	9	4	8	12	9
Demand		3	3	4	5	6	21

6. Solve the travelling-salesman problem given by the following data
   $C_{12} = 20,\quad C_{13} = 4,\quad C_{14} = 10,$
   $C_{23} = 5,\quad C_{34} = 6$
   $C_{25} = 10,\quad C_{35} = 6,\quad C_{45} = 20,$
   $where\ C_{ij} = C_{ji}$ and there is no route between cities I and j if the value of C_{ij} is not shown.

Section-D

7. Solve the following (2 X 4) game:

		B			
		I	II	III	IV
A	I	2	2	3	-1
	II	4	3	2	6

8. Solve the following Linear Programming problem by dynamic programming approach.
   Maximize $z = 2x_1 + 5x_2$, subject to constraints
   $2x_1 + x_2 \leq 43,\ 2x_2 \leq 46\ and\ x_1 \geq 0, x_2 \geq 0$

Exam Code: 225703 (20)

Paper Code: 3216

Programme: Master of Science (Mathematics)
Semester-III

Course Title: Statistics-I

Course Code: MMSL- 3334

Time Allowed: 3 Hours

Max Marks: 70

Note: Candidates are required to attempt five question selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 14 marks. The students can use only non-Programmable & Non-Storage Type Calculator.

Section-A

1. (a) What are the criteria of a good measure of central tendency? Also, Prove that the sum of the squared deviation is least when taken from the mean.

   (b) What do you mean by skewness and kurtosis of a distribution? Describe briefly their measures.

2. (a) Give various approaches to define probability. Write limitations of each, if any.

b) Let X has the probability density function

$$f(x) = \begin{cases} c + x \; ; & -2 < x \leq 0 \\ c - x \; ; & 0 < x \leq 2 \\ 0 \; ; & otherwise \end{cases}$$

Determine the constant c, cumulative density function $F\,(x)$ and find P $\left(-\frac{1}{3} < x \leq \frac{4}{3}\right)$.

Section-B

3. a) A man with n keys wants to open his door and tries the keys independently and at random. Find the mean and variance of number of trials required to open the door, if unsuccessful keys are

(i) Not eliminated from further selection and

(ii) Eliminated from further selection.

b) Let X be a non-negative random variable with distribution function F, show that:

$$E\,(X) = \int_0^\infty [1 - F(x)]\,dx - \int_{-\infty}^0 F\,(X)dx$$

4. a) Let (X,Y) be two-dimensional non-negative continuous random variables having the joint density function:

$$f(x,y) = \begin{cases} 4xy\, e^{-(x^2+y^2)} & ; x \geq 0, y \geq 0 \\ 0 & ; otherwise \end{cases}$$

Find the density functions of U = $\sqrt{X^2 + Y^2}$

b) Discuss the properties of moment generating function along with their limitations.

Section-C

5. a) State Chebyshev's Inequality and show that it leads to the weak law of large numbers.

b) If $X \sim B(n, p)$, obtain the recurrence relation for its central moments and hence obtain values of β_1 and β_2

6. a) Define geometric distribution of a random variable. Obtain its mean and variance. Also prove the lack of memory property of it.

b) A manufacturer, who produces medicine bottles, finds that 0.1% of bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain

(i) no defective, and

(ii) at least two defectives.

Section-D

7. a) What is normal distribution? Describe its important properties.

b) If X and Y are independent Gamma variate with parameters μ and v respectively, then show that Z= X+Y is also distributed as gamma with parameter $(\mu + v)$.

8. a) What is association of attributes? Write a note on the strength of association and how it is measured.

b) Obtain the equation of two lines of regression for the following data. Also obtain the estimate of X for Y=70

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71