Exam Code: 225701 Paper Code: 1219 - R

Programme: Master of Science (Mathematics) Semester-I

Course Title: Real Analysis

Course Code: MMSL – 1331

Time Allowed: 3 Hours **Max. Marks: 70**

Note: Attempt five questions in all, selecting atleast one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

Section – A

Q-1 (a) Prove that set of all rational numbers is countable.

(b) Define the Space R^3 and prove that it is a metric Space under the metric d defined as

$$[d(x,y)]^2 = \sum_{i=1}^{3}(x_i - y_i)^2 \ \forall \ x,y \in R^3$$

Q-2 (a) Find open and closed spheres in the plane R^2. Also find the derived set of set of natural numbers and set of irrational numbers.

(b) Prove t generalized Heine Borel theorem. Also prove that Cantor's set is a perfect set.

Section – B

Q-3 (a) Prove that union of two closed sets is a closed set with respect to the condition that A and B are Separated sets. Also find out all the components of a discrete metric space.

(b) Prove that a metric Space is disconnected(X,d) iff there exist a non empty proper subset of X which is open as well as closed.

Q-4 (a) Prove that diam($\bar{E}$)= diam(E) . Also prove that the set C of complex numbers with usual metric space is complete metric space.

(b) State and Prove Cantor's intersection theorem.

Section – C

Q-5 (a) State and Prove Baire's Catagory theorem

(b) State And Prove fixed point threorem . Also give illustrative example.

Q- 6 (a) Define uniform continuity in a metric space and establish a relation between continuity and uniform continuity

(b) if f is mapping from X to Y. Then f is continuous iff image of the closure of A is contained in closure of the image of A

Section – D

Q-7 (a) State and prove W.M. Test for uniform Convergence of Series.

(b) Prove that the series $\sum(-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval but does not converge absolutely for any value of x.

Q-8 (a) State and Prove lemmas of Weierstrass Approximation theorem.

(b) If $f_n(x) = \frac{x^2}{x^2+(1-nx)^2}, 0\leq x \leq 1$, n=1,2,3…., then prove that given sequence is uniformly bounded on [0,1] , no subsequence of $f_n(x)$ can converge uniformly on [0,1] and the given sequence is not equicontinuous on [0,1]

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Section – A

Q-1 **(a)** Let X=C[0,1]=set of all valued continuous functions defined on [0,1] . Consider a mapping d: $X\times X \rightarrow R$ defined by d(f,g)= Sup $\{|f(x)-g(x)|:$ x∈ [0,1]} $\forall f, g \in X$. Then prove that d is metric on C [0,1].

(b) Prove that set of integers is countable.

Q-2 **(a)** Let (X, d) be a metric Space and E ⊂ X *be* any set, $\overline{E}$ is closed set and it is the smallest closed set containing set E.

(b) Define K-cell and Prove that every k-cell is compact.

Section – B

Q-3 **(a)** Prove that Union of any family of connected sets having non-empty intersection is a connected set and every component is a closed set.

(b) Prove that intervals and only intervals are connected subset of R.

Q-4 (a) Prove that a metric space is compact iff any sequence in X has a convergent subsequence.

(b) State and Prove Cantor's Intersection Theorem.

Section – C

Q-5 (a) Prove that every Complete Metric Space cannot be expressed as union of a sequence of nowhere dense sets.

(b) Define fixed point of a mapping .Prove that every contraction mapping is continuous. What are convergent sequences in a discrete metric space?

Q-6 (a) Prove that continuous image of a compact metric space is compact. Also give an example to show that Continuous image of open set need not open.

(b) Prove that monotone functions have no discontinuities of second kind. Also state intermediate value theorem.

Section – D

Q-7 (a) State and Prove theorem on Term by Term Integration of uniform convergence of the series.

(b) Prove that the series converges $\sum \frac{\sin nx}{n}$ uniformly w.r.t x in $[\alpha, 2\pi - \alpha]$ *where* $0 < \alpha < \pi$.

Q-8 (a) State and Prove W. M test for uniform convergence of the series.

(b) Test the convergence of the series

$$\sum_{n=1}^{\infty}\left\{ \frac{2\,n^2x^2}{e^{(nx)^2}} - \frac{2(n-1)^2x^2}{e^{((n-1)x)^2}} \right\}$$

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Exam Code: 225701 (20)

Paper Code: 1220

Programme: Master of Science (Mathematics)
Semester-I

Course Title: Complex Analysis

Course Code: MMSL-1332

Time Allowed: 3 Hours

Max Marks: 70

Note:- Attempt five questions, selecting at least one question from each section. The Fifth question may be attempted from any section. Each question carries 14 marks.

Section-A

1. (a) Examine the nature of the function prove that

$f(z) = \frac{x^2 y^5 (x+iy)}{x^4+y^{10}}$, $z \neq 0$, f(0)=0 in the region including the origin. 7

(b) Show that $u = \frac{1}{2}\log\,(x^2 + y^2)$ is harmonic and find its harmonic conjugate. 7

2. (a) Using Cauchy' s Integral formula, Calculate

$\int_c \frac{1}{z(z+\Pi i)} dz$, Where C the circle $|z + 3i| = 1$. 7

(b) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is analytic function of $z = x + iy$, find $f(z)$ in terms of z. 7

Section-B

3. (a) Use the transformation w=$\frac{1}{z}$, find the image of |z-2i| =2. 7

(b) Evaluate $\int_c \frac{1}{z}\, dz$, where C denotes the square described in the positive sense with sides parallel to the axis and of length $2a$ and having its centre at the origin. 7

4. (a) Find the Mobius transformation which maps 1,-i,2 onto 0,2,-i, respectively. 7

(b) Find a bilinear transformation which maps the |z| = 1, i,-1 respectively onto w= i, 0,- i and w=1, for this transformation, find the images of respectively into z = $\frac{1}{2}$, z=0. 7

Section-C

5. State Taylor's theorem and Laurent's theorem and obtain the Taylor's and Laurent's series which represent the function $f(z) = \frac{z^2-1}{(z+3)\,(z+2)}$, When

(i) $|z|<2$ (ii) $2<|z|<3$ (iii) $|z|>3$

14

6. (a) Find the residue of $f(z) = \frac{z^3}{(z-1)(z-2)(z-3)}$, at z= 1,2,3 and ∞ . And show that the sum of these residues is zero. 7

(b) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z|=1$ and $|z|=2$.

7

Section-D

7. (a) Show that $z = a$ is an isolated essential singularity of the function $\frac{e^{\frac{c}{z-a}}}{e^{\frac{c}{z}}-1}$. 7

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2}$, where $a > 0, b > 0, a > b$.

7

8. (a) Show that $\int_0^{\pi} \frac{1+2\cos\theta}{(5+4\cos\theta)} d\theta = 0$. 7

(b) Show by Contour integration that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$.

7

Exam Code: 225701 (20) Paper Code: 1221

Programme: Master of Science (Mathematics)
Semester-I

Course Title: Algebra-I

Course Code: MMSL-1333

Time Allowed: 3 Hours **Max Marks: 70**

There are total Eight questions of equal marks (14 marks each), two in each of the four Sections (A-D). Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any Section.

Section A

1. (a) Prove that union of two subgroups of a group is a subgroup iff one is contained in the other. 7

   (b)State and prove Lagrange's theorem for finite groups. Is its converse true. 7

2. (a) Prove that every subgroup of a cyclic group is cyclic. 7

   (b) Show that every subgroup of an abelian group is normal. 7

Section B

3. (a) For a finite group G, prove that $0(G) = \sum \frac{0\,(G)}{0(N(a))}$ where the sum runs over one element a of each conjugate class. 7

   (b) State and prove Cayley's Theorem. 7

4. (a) Prove that A_4 has no subgroup of order 6. 7

   (b) Prove that the group of automorphisms of a cyclic group is abelian. 7

Section C

5. (a) State and prove Sylow's First theorem. 7

   (b) Every finite group has atleast one composition series. 7

6. (a) State and prove Jordan Holder theorem. 7

   (b) Show that every group of order 15 is cyclic. 7

Section D

7. (a) Define internal direct products. 7

   (b) Show that the group Z_8 cannot be written as the direct sum of two non-trivial subgroups. 7

8. (a) Show that the direct product of two cyclic groups G and G′ is a cyclic group iff $(0(G)\ 0(G'))\ = 1$ 7

   (b) Prove that Klein's four group is the direct product of its subgroups. 7

Exam Code: 225701 **Paper Code: 1222**

Programme: Master of Science (Mathematics)

Semester – I

Course Title: Mechanics-I

Course Code: MMSL-1334

Time Allowed: 3 Hours **Max. Marks: 70**

Note: Attempt five questions in all, selecting atleast one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

Section-A

Q1 (a) Show that two equal and opposite rotations of a rigid body about distinct axes are equivalent to a translation of the body. **(5)**

(b) A cyclist rides along a straight road with uniform speed V. At a given instant, the cyclist is at a point O on the road and the dog is at A , a point distant 'a' from the road and such that OA is perpendicular to the road. The dog runs with speed V towards the cyclist . Taking OA and the road as axes of x and y respectively and coordinates of the dog as (x,y).

Show that $[\, x^2 (\frac{d^2y}{dx^2})^2]=\left[1 + (\frac{dy}{dx})^2\right]$ and hence prove that the equation of dog's path is $y = \frac{1}{2} \{ (\frac{x^2}{2a}) - a \log(\frac{x}{a}) - \frac{1}{2} a \}$. **(9)**

Q2 (a) Write a short note on hodograph. **(5)**

(b) A point P moves so that it always has a velocity compounded of V_1 parallel to a fixed line and V_2 in the transverse direction perpendicular to OP , where O is a fixed point. Prove that the orbit is a conic having O as a focus and discuss the cases when $V_1 > V_2$, $V_1 < V_2$, $V_1 = V_2$. **(9)**

Section-B

Q3 (a) A particle of mass m oscillates in a line with natural period $\frac{2\pi}{n}$. If an applied periodic force F Cos pt now acts in the line so that the particle is instantaneously at rest at zero time at a distance d from the center of oscillation , prove that the displacement of the particle at a subsequent time t is

$$d \cos nt + \frac{F(\cos pt - \cos nt)}{(n^2 - p^2)m}$$ **(7)**

(b) Discuss in detail simple harmonic motion. **(7)**

Q4 Discuss the motion of a particle in a cycloid. **(14)**

Section-C

Q5 (a) Derive the differential equation of the orbit moving under a central force P per unit mass. **(7)**

(b) A particle is describing an ellipse of eccentricity e about a centre of force at a focus. Prove with usual notations $V^2 = \mu[2/r - 1/a]$, $h^2 = \mu a(1 - e^2)$ When the particle is at one end of a minor axis its velocity is doubled. Prove that the new path is a hyperbola of eccentricity $(9 - 8e^2)^{1/2}$. **(7)**

Q6 (a) A particle of mass m describes the curve $r = a/(1+2\cos^2\theta)$ under a force to the origin O. If the speed of the particle at the farther apse is v, find the law of force. When passing A, one of the nearer apses, the particle receives an impulse mkv along the outward normal. . Show that if $k < 2\sqrt{3}$ the particle continues to describe a closed curve about the origin, but if $k > 2\sqrt{3}$ the new orbit has one asymptote making an angle θ with OA given by $\cos(\alpha+2\theta) + 2\cos\alpha = 0$ where $\tan\alpha = \frac{1}{2}k$ **(11)**

(b) A particle of mass m is moving in a plane under a force F which is always directed towards the fixed point O. Show that $r^2\dot{\theta} \doteq 0$. **(3)**

Section-D

Q7 (a) Determine the moment of inertia of the distribution about the axis through O having direction cosines [λ, μ , v] in terms of these direction cosines and A,B,C,D,E,F. **(9)**

(b) Find the moment of inertia and product of inertia of an elliptic disc. **(5)**

Q8 (a) Define Principal Axes. Show that at each point of a body there are three principal axes mutually at right angles to each other such that the product of inertia about them taken two at a time is zero. **(9)**

(b) State and prove the theorem of perpendicular axes for moments of inertia. **(5)**

Exam Code: 225701 Paper Code: 1223

Programme: Master of Science (Mathematics) Semester-I

Course Title: Mathematical Programming-I

Course Code: MMSL-1335

Time Allowed: 3 Hours Max Marks: 70

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

Section-A

1. a) Explain the terms unbounded solution, infeasible solution. 4

b) Use two phase simplex method to maximize $z = 5x_1 + 3x_2$

subject to the constraints: $2x_1 + x_2 \leq 1\,; x_1 + 4x_2 \geq 6$

$x_1, x_2 \geq 0$

10

2. a) Obtain the basic solutions to the following system of linear equation; 4

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

b) Use simplex method to solve the following LPP maximize $z = 4x_1 + 10x_2$ subject to the constraints:

$$2x_1 + x_2 \leq 50;\ 2x_1 + 5x_2 \leq 100;\ 2x_1 + 3x_2 \leq 90$$
$$x_1 \geq 0, x_2 \geq 0$$

10

Section-B

3. a) State and prove Complementary Slackness theorem. 4

b) Find the starting solution in the following transportation problem by Least Cost Method.
Also obtain the optimum solution by using the best starting solution

	D_1	D_2	D_3	D_4	Supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	**3**	**3**	**2**	**2**	

10

4. a) Formulate the dual of the following linear programming problem:

Maximize $z = 5x_1 + 3x_2$ subject to the constraints:

$3x_1 + 5x_2 \le 15;\ 5x_1 + 2x_2 \le 10$

$x_1 \ge 0, x_2 \ge 0$ 4

b) If the matrix elements represent the unit transportation times, solve the following transportation problem:

From	To				Available
	D_1	D_2	D_3	D_4	
O_1	10	0	20	11	**15**
O_2	1	7	9	20	**25**
O_3	12	14	16	18	**5**
Required	**12**	**8**	**15**	**10**	**45**

10

Section-C

5. a) The following is the cost matrix of assigning 4 clerks to 4 key punching jobs. Find the optimal assignment if clerk 1 cannot be assigned to job 1:

Clerk	**Job**			
	1	**2**	**3**	**4**
1	--	5	2	0
2	4	7	5	6
3	5	8	4	3
4	3	6	6	2

What is the minimum total cost? 10

b) Consider payoff matrix with respect to Player A and Solve it optimally:

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 6 & 9 \\ 8 & 4 \end{bmatrix} \end{array}$$

4

6. a) Give the mathematical formulation of an assignment problem. 4

b) Solve the following 3 X 5 game using dominance property

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 2 & 5 & 10 & 7 & 2 \\ 3 & 3 & 6 & 6 & 4 \\ 4 & 4 & 8 & 12 & 1 \end{bmatrix} \end{array}$$

10

Section-D

7. a) Find the optimum integer solution to the following linear programming problem. Maximize $Z = 5x_1 + 8x_2$ Subject to

$$x_1 + 2x_2 \leq 8;\ 4x_1 + x_2 \leq 10$$
$$x_1 \geq 0, x_2 \geq 0 \text{ and integers}$$

10

b) Explain the disadvantages of dynamic programming. 4

8. a) What are the applications of integer programming problem? 4

b) Solve the following LPP using dynamic programming technique:

Maximize $Z = 10x_1 + 30x_2$ Subject to

$$3x_1 + 6x_2 \leq 168;\ 12x_2 \leq 240$$
$$x_1 \text{ and } x_2 \geq 0$$

10