Exam Code: 508405 Paper Code: 5175
(20)

Programme: Master of Science (Mathematics) (FYIP)
Semester-V

Course Title: Number Theory

Course Code: FMAL-5331

Time Allowed: 3 Hours Max Marks: 80

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any Section. All questions carry equal marks.(16 marks each).

Section-A

1. (a) Find all positive solutions in integers for $7x+19y=213$. 8

   (b) Prove that an integer is divisible by 4 if and only if the number formed by its last two digits is divisible by 4. 8

2. (a) Find the smallest integer $a > 2$, such that $2 | a, 3 | (a+1), 4 | (a+2), 5 | (a+3), 6 | (a+4)$. 8

(b) Check whether {-27,-18,10,17,20,49} is a complete residue system (mod 6). 8

Section-B

3. (a) Prove that the linear congruence $ax \equiv b \ (mod\ m)\ has\ a\ solution\ if\ and\ only\ if\ d|b\ where\ d = \gcd(a, m)$.

8

(b) Solve the system of given congruences: $x \equiv 3\ (mod\ 13), x\ \equiv 7\ (mod\ 11), x\ \equiv 5\ (mod\ 7)$ 8

4. (a) Check whether 1729 is a pseudo prime or not. 8

(b) State and prove Fermat's theorem. Is its converse true? Justify your answer. 8

Section-C

5. (a) Let p denotes a prime. Then using Wilson's theorem check whether $x^2 \equiv -1(mod\ p)$ has solution if $p = 2\ or\ p \equiv 1\ (mod\ 4)$. 8

(b) Check whether for any positive integer n,$\emptyset\ 3(n) = 3\emptyset(n)\ if\ f\ 3|n$. 8

6. (a) Using Euler theorem, prove that $3^{500} \equiv 1(mod\ 100)$. 8

(b) Verify that $\sum_{d|36} \emptyset(d) = 36$. 8

Section-D

7. (a) Prove that the sum function $\sigma(n)$ is a multiplicative function. 8

(b) Check whether $\prod_{d|n} d = n^{\frac{\tau(n)}{2}}$. 8

8. (a) For every positive integer n having k distinct prime factors $\sum_{d|n} \mu(d)\tau(d) = (-1)^k$. Prove or disprove above statement. 8

(b) By using Mobius Inversion formula, find the value of $\emptyset(n)$. 8

Exam Code: 508405 (20)

Paper Code: 5176

Programme: Master of Science (Mathematics) (FYIP)
Semester-V

Course Title: Discrete Mathematics

Course Code: FMAL-5332

Time Allowed: 3 Hours

Max Marks: 80

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries 16 marks.

Section-A

1. a) Let A be the set of positive factors of 70. Two binary operations '+' and '.' are defined as follow:

$a + b = l.c.m\,(a, b) and\; a.b = g.c.d\,(a, b) \forall a, b \in A.$

A unary operation '.' on A defined as a' = $\frac{70}{a}$ $\forall a \in A$. Is (A,+,., ') a Boolean algebra?

b) Consider the Boolean expression $f_1\,(a.b.c) = ((ab)'c)'((a' + c)(b' + c'))'$ into a sum-of-product form.

(10,6)

2. a) Consider the Boolean function $f\,(a, b, c) = a'b(a' + c) + ab'(b' + c)$, Solve it algebraically.

b) Simplify f (in part(a)) with the help of karnaugh Map.

c) Draw the switching (on-off) circuit of f and the reduction of f obtained in part (a).

d) Draw the circuit (gate) diagram of the reduction of f obtained in (b) . (4,4,4,4)

Section-B

3. a) How many nodes are necessary to construct a 2 regular graph with exactly 6 edges?

   b) Define the union and intersection of two graphs with an example.

   c) Let G=(V,E) be a simple connected planar with more than one edges then following inequality holds:

   (i) $2|E| \geq 3$ R

   (ii) $|E| \leq 3\,|V| - 6$

   (iii) There exist a vector 'v' of G such that deg $(v) \leq 5$

   (4,4,8)

4. a) Define the Euler circuit and Hamiltonian Circuit, give an example of a graph which is Euler circuit but not Hamiltonian circuit.

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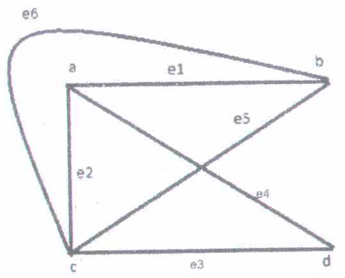
b) Find the Euler path or circuit (if exist) in the following graph



c) Explain the Konigsberg problem with the help of Graph.

d) Prove that If a graph has an Euler path then it has exactly two vertices of odd degree. Is the converse true, if not then under what condition the converse is true. Explain it.

(3,3,5,5)

Section-C

5. a) Let G= (V,E) be an undirected graph with no self loops and |V|=n. The following are all equivalent:

(i) G is a tree.

(ii) For each pair of distinct vertices in V there exists a unique simple path between them.

(iii) G is connected and if e $\in E$, then (V,E-$\{e\}$) is disconnected.

(iv) G contains no cycle but by adding one edge, you create a cycle.

(v) G is connected and E=n-1

(vi) Prove that maximum no. of vertices on level n of a binary tree is 2^n (10,6)

6. a) Draw the unique binary tree. Given that

In-order	q	b	c	a	g	p	e	d	r
Pre-order	g	b	q	a	c	p	d	e	r

(b) Define Spanning tree and Give all spanning trees of K_4.

(8,8)

Section-D

7. a) What are the fundamental connectors. Explain it with the help of an example.

b) Check whether that $(p \uparrow q) \oplus (p \uparrow q)$ is equivalently to $(p \vee q) \wedge (p \downarrow q)$ or not? (8,8)

8. a) Check whether that $pv \sim (p \wedge q)$ is tautogy or not?

(8,8)

b) Determine the validity of the following argument:

Either I will pass the examination or I will not graduate. If I do not graduate, I will go to Canada. I failed : Thus, I will go to Canada (8,8)

Exam Code: 508405 **Paper Code: 5177**
(20)

Programme: Master of Science (Mathematics) (FYIP)
Semester-V

Course Title: Linear Integral Equations

Course Code: FMAL-5333

Time Allowed: 3 Hours **Max Marks: 80**

Note: There are total Eight question of equal marks (16marks each), two in each of the section (A-D). Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any Section.

Section-A

1. (a) Show that $F(x) = cos2x$ is a solution of the equation

$$F(x) = cosx + 3\int_0^{\pi} K\,(x,\eta)F(\eta)\,d\eta$$

$$Where\ K\,(x,\eta) = \begin{Bmatrix} sinx\ \cos\eta, & 0 \le x \le \eta \\ cosx\ sin\eta, & \eta \le x \le \pi \end{Bmatrix}$$

8

b) Obtain the resolvent kernels of the integral equation with the following kernel $\lambda=1$, $k(x,\eta)=2x$.

8

2. a) Solve the integral equation

$F(x) = (1 + x) + \lambda \int_0^x (x - \eta)F(\eta)\, d\eta$ 8

b) Solve $y(x) = 29 + 6x + \int_0^x [5 - (x - t)]y(t)dt.$

8

Section-B

3. a) Solve the integral Equation $g(s) = f(s) + \lambda \int_0^{2\pi} \sin s.\ cost\ g(t)dt$ to a resolvent kernel. 8

b) Define

(i) Symmetrical kernel

(ii) Algebraic multiplicity of eign value. 8

4. a) Find the eign values and eign functions of the homogeneous integral equation

$$F(x) = \lambda \int_0^{\pi} (cos^2x\ cos2\eta + cos3x\ cos^3\eta)F(\eta)\, d\eta$$

8

b) Solve the integral equation by the method of successive approximations $y(x) - \frac{5x}{6} + \frac{1}{2}\int_0^1 xt\, y(t)\, dt.$ 8

Section-C

5. a) Solve by the method of Laplace transform

$3sin2x = y(x) + \int_0^x (x-t)y(t)dt$ 8

b) Solve the integral equation

$$F(t) = t^2 + \int_0^t F(u)\sin(t-u)du$$

8

6. a) Solve the integral equation $t = \int_0^t e^{(t-u)}F(u)du$ 8

b) Solve the Abel's integral equation

$\int_0^t \frac{F(u)du}{\sqrt{(t-u)}} = 1 + \mathrm{t} + t^2$ 8

Section-D

7. a) Define Green's function and discuss its four properties. 8

b) Reduce the boundary value problem $y'' + \lambda y = x, y\,(0) = y\left(\frac{\pi}{2}\right) = 0$ to an integral equation. 8

8. a) Reduce the boundary value problem $y'' + xy = 1, y\,(0) = 0, y(l) = 1$ an integral equation. 8

b) Construct Green's function for the homogeneous boundary value problem $\frac{d^4y}{dx^4} = 0, y(0) = y'(0) = y(1) = y'(1) = 0$ 8

Exam Code: 508405 (20)

Paper Code: 5178

Programme: Master of Science (Mathematics) (FYIP)
Semester-V

Course Title: Riemann Integration

Course Code: FMAL-5334

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any Section. Each question carries 16 marks.

Section-A

1. (a) Define Norm of a partition. 02

   (b) *Let* f be a bounded function defined on [a, b]. If P_1 and P_2 are any two partitions of [a, b]. Then U $(P_2, f) \geq L(P_1, f)$ 07

   (c) Prove that lower R-integral cannot exceed the upper R-integral. 07

2. (a) Show that $f(x) = sinx$ is integrable on $\left[0, \frac{\pi}{2}\right]$ and $\int_0^{\frac{\pi}{2}} \sin x \ dx = 1.$ 08

(b) State the prove Darboux Theorem. 08

Section-B

3. (a) Every continuous function is integrable. 08

(b) If f is bounded and integrable in [a, b] then |f| is also bounded and integrable in [a, b] and $|\int_a^b f(x)\,dx| \leq \int_a^b |f(x)|dx.$ 08

4. (a) Let $f \in R$ [a, b]. Then prove that the function defined on [a, b] by $F(x) = \int_0^x f(t)dt$ is continuous. 08

(b) If f is continuous in [a, b] then there exists a number c, lying between a and b such that $\int_a^b f(x)dx = (b-a)f(c)$ 08

Section-C

5. (a) Show that Beta function $\int_0^1 x^{(m-1)}(1-x)^{(n-1)}\,dx$ converges iff m>0 and n>0. 08

(b) Show that $\int_0^{\frac{\pi}{2}} \log\sin x\, dx$ is convergent and hence evaluate it. 08

6. (a) Show that $\int_0^\infty \sin x^2 \, dx$ is convergent. 08

(b) If $\int_0^\infty f(x)dx$ is convergent at ∞ and $\varphi(x)$ is bounded and monotonic for $x \geq a$ then $\int_a^\infty f(x)\varphi(x)dx$ is convergent at ∞. 08

Section-D

7. (a) Prove that every absolute convergent integral ~~in~~ is convergent. 06

(b) If n is a positive integer, prove that

$2^n \, \Gamma\left(n + \frac{1}{2}\right)$ = 1.3.5 ... $(2n-1)\sqrt{\pi}$. 06

(c) Evaluate $\int_0^\infty x^6 \, e^{-2x} \, dx$. 04

8. (a) Prove that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \, m > 0, n > 0.$ 08

(b) Prove that $\int_0^{\frac{\pi}{2}} (\tan x)^n \, dx = \frac{\pi}{2} \sec \frac{n\pi}{2}$, $-1<n<1$ 04

(c) If n is a non- negative integer, then Γ (n+1)=n! 04

Exam Code: 508405 (20) Paper Code: 5179

Programme: Master of Science (Mathematics) (FYIP)
Semester-V

Course Title: Metric Spaces

Course Code: FMAL-5335

Time Allowed: 3 Hours Max Marks: 80

Note : Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section, Each question carries 16 marks.

Section — A

1. (a) Let X be a non empty set. Consider a mapping $d: X \times X \to R$, defined by

$$d(x,y) = \begin{cases} 0, when\ x = y \\ 1, when\ x \neq y \end{cases} \quad \forall\, x, y \in X$$

Prove that it d is a metric on X.

(b) Prove that every metric space is pseudo metric space but converse may not be true. (8,8)

2. (a) Prove that interior of any subset of A of metric space (X, d) is largest open subset of A,

(b) Let (R, d) be a usual metric space.

Find:-

(i) Interior

(ii) Exterior of the following sets :

a) N

b) (0,1)

(8.8)

Section — B

3. (a) Prove that closed subset of compact set are compact.

(b) Is Usual metric space (R,d) compact or not?

(10,6)

4. (a) State and prove Heine Borel theorem.

(b) Prove that a metric space (X ,d) is disconnected iff there exist a non empty proper subset of X which is both open as well as closed (8,8)

Section — C

5. (a) Prove that every convergent sequence is bounded but converse is not true.

(b) Prove that the set of complex numbers with usual metric space is complete metric space

(8,8)

6. State and prove Cantor's intersection theorem

(16)

Section — D

7. (a) Let $(X, d_1)$and (Y, d_2) be two-metric space. Prove that A function $f: X \rightarrow Y$ is continuous iff $f^{-1}(G)$ is open set in X for every open set G in Y.

(b) Give an example to show that the continuous image of an open set need not to be open. (8,8)

8. (a) Is every continuous function a uniform continuous ? If not then under what conditions a continuous function is uniformly continuous.

(b) State and prove Intermediate Value Theorem.

(10,6)