Exam Code: 508403 Paper Code: 3198

Programme: Master of Science (Mathematics) (FYIP)

Semester-III

Course Title: Graph Theory

Course Code: FMAL-3330

Time Allowed: 3 Hours Max. Marks: 70

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

Section-A

Q.1 (a) Justify how many vertices are necessary to construct a 3 regular graph with exactly 6 edges ? Also draw the graph. (7)

(b) Define subgraph and homeomorphic graph with an example. Also discuss the union and intersection of two graphs with an example. (7)

Q.2 (a) Prove or disprove that a graph G is self complementary if it has 4n or 4n+1 vertices (n is a non negative integer). (7)

(b) Define complete graph and planar graph with an example. Justify whether or not a complete graph K_5 is non planar. (7)

Section-B

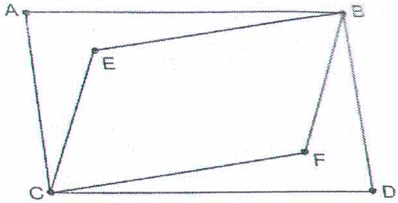
Q.3 (a) Prove or disprove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (7)

(b) Let G=(V,E) be a simple connected planar graph with more than one edges and R regions, then justify whether or not the following inequality holds :

$(i)\ 2|E| \geq 3R$ $\qquad (ii) |E| \leq 3\,|V| - 6$

$(iii)\ There\ exist\ a\ vertex\ 'v'\ of\ G\ \ such\ that\ \deg(v) \leq 5$ (7)

Q.4 (a) Define Euler and Hamiltonian Circuit. Does the graph given below has Euler or Hamilton Circuit. If yes or no, then justify your answer by giving proper steps.



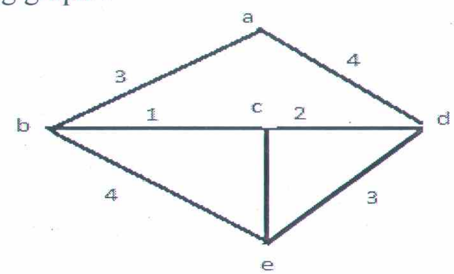
(7)

(b) Explain the Konigsberg problem with the help of Graph. (7)

Section-C

Q.5 (a) Justify whether or not that in any non-trivial tree, there are atleast two pendent vertices. (7)

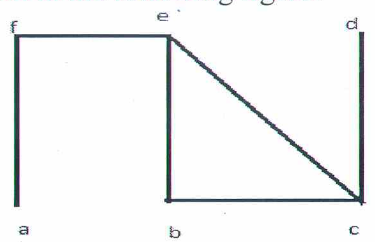
(b) Using Kruskal's algorithm , find minimal spanning tree for the following graph :



(7)

Q.6 (a) Define m-ary tree with an example. Represent the expression as a binary tree and write the prefix and post fix forms of the expression$\big((a+b)\uparrow 5\big)+((a-b)/2)$. (7)

(b) Define spanning tree with an example. Find all spanning trees of the graph shown in the following figure:



(7)

Section-D

Q.7 (a) Prove or disprove that chromatic number of $K_{m,n}$ is 2. (7)

(b) Justify whether or not that every planar graph is five colourable. (7)

Q.8 (a) Prove or disprove that a graph G is complete graph if and only if chromatic number of G is n. (7)

(b) Draw the directed graph G whose incidence matrix is as under

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}.$$ (7)

Exam Code: 508403 Paper Code: 3194

Programme: Master of Science (Mathematics) (FYIP)

Semester – III

Course Title: Theory of Real Functions

Course Code: FMAL-3331

Time Allowed: 3 Hours Max. Marks: 70

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

SECTION-A

1. (a) Define limit of a function. If $\lim_{x\to a} f(x)=l$ and $\lim_{x\to a} g(x)=l$ then prove that $\lim_{x\to a}(fg)(x)=lm$. 7

(b) Using $\in-\delta$ definition of limit, prove that $\lim_{x\to 2}(5x-3)=7$. 7

2. (a) Let $f(x)=\begin{cases}1 & ; x\le 3\\ ax+b & ; 3<x<5\\ 7 & ; 5\le x\end{cases}$. Determine the constants a and b so that $f(x)$ may be continuous for all x. 8

b) State discontinuity criterion. Explain the types of discontinuity. 6

SECTION-B

3. (a) Let $f(x)=\begin{cases}2x+1 ; & 0\le x\le 1\\ 2+x ; & 1<x\le 5\end{cases}$ and $g(x)=\begin{cases}x^2 & ; 0\le x\le 1\\ 3x-1 ; & 3<x\le 5\end{cases}$. Find all points of discontinuity of $g\,o\,f$ in $[0,2]$. 7

(b) Prove that $f(x)=x^3+1$ is uniformly continuous in $[0,1]$. 7

4. (a) If f and g are two differentiable functions on an interval I, then prove that $f+g$ is also differentiable on the interval I, and $(f+g)' = f' + g'$ 7

(b) Examine the differentiability of $f(x)$ at $x=2$, where

$$f(x) = \begin{cases} x-1 & ; x<2 \\ 2x-3 & ; x \geq 2 \end{cases}$$ 7

SECTION-C

5. (a) Find the values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Also find the points of inflexion. 7

(b) Use mean value theorem to prove that

$$1 + x \leq e^x \leq 1 + xe^x \ \forall\ x \geq 0$$ 7

6. (a) Evaluate the limit $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ using L' Hospital Rule. 7

(b) State and prove Cauchy's Mean value theorem. 7

SECTION-D

7. State and prove Taylor's theorem with Lagrange's form of remainder. Also, state Maclaurin's theorem with Lagrange's form of remainder. 14

8. (a) Expand $\sin x$ in ascending powers of $\left(x - \frac{\pi}{2} \right)$. 7

(b) Use Maclaurin's theorem to show that

$$e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots + \frac{x^{n-1}}{\lfloor n-1} + \frac{x^n}{\lfloor n} e^{\theta x} ; 0 < \theta < 1.$$ 7

Exam Code: 508403 Paper Code: 3195

Programme: Master of Science (Mathematics) (FYIP)

Semester: III

Course Title: Applied Differential Equations

Course Code: FMAL-3332

Time Allowed: 3 Hours Max. Marks: 70

Note: Attempt five questions in all selecting atleast one question from each section. Fifth question can be attempted from any section. Each question carries 14 (7+7) marks. Students are allowed to use scientific non programmable Calculator.

Section - A

Q1. (a) What are the classifications, origin and application of Differential Equations?

(b) Show that every function f defined by f(x) = $(x^3 + c)e^{-3x}$, where c is an arbitrary constant, is a solution of the differential equation

$\frac{dy}{dx} + 3y = 3x^2e^{-3x}$ [7,7]

Q2. (a) Solve the initial value problem

(2ysinx cos x + y^2sinx)dx + ($sin^2x - 2ycosx$)dy = 0,y(0) = 3

(b) Solve the differential equation by separating the variables

x sinydx+(x^2+1)cosydy=0, y(1)=$\pi/2$ [7,7]

SECTION - B

Q3. (a) Find the orthogonal trajectories of the family of circles which are tangent to the y axis at the origin

(b) Find a family of oblique trajectories that intersect the family of curves x + y = cx² at an angle αs.t tanα = 2 [7,7]

Q4. (a) Explain the limited growth Model in detail. Derive its differential equation and obtain its solution. Also sketch the graph and describe how the population approaches the carrying capacity as time increases.

(b) Explain the process of assimilation of a single cold pill into the bloodstream. Describe each stage and represent the equation of drug concentration in blood using an exponential model and a well labelled graph.

[7,7]

SECTION-C

Q5. (a) Solve the non-homogeneous differential Equations with constant Coefficients

$$y'' - 4y' + 4y = 8(x^2+e^{2x}+\sin 2x)$$

(b) Solve the differential Equations

$$\frac{d^4y}{dx^4} - y = x^2 \sin x$$

[7,7]

Q6. (a) Solve the Cauchy Euler equations w.r.t initial condition

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3,\ y(2)=0,\ y'(2) = -8$$

(b) Find the general solution of the following non-homogeneous differential equations using method of variation of parameters.

$$(D^3 - 6D^2 + 11D - 6)y = e^x$$

[7,7]

SECTION-D

Q7. (a) Solve in series

$$(2x+x^3)\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6xy = 0$$

(b) Solve Hermite Differential Equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2ny = 0$

[7,7]

Q8. (a) Solve in series by method of Frobenius

$$4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

(b) Solve Bessel Equation of order zero,

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-1)y = 0$$

[7,7]

Exam Code: 508403 **Paper Code: 3196**

Programme: Master of Science (Mathematics) (FYIP)

Semester-III

Course Title: Logic and Reasoning

Course Code: FMAL-3333

Time: 3 Hours **Max. Marks: 70**

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

Section-A

Q 1 (A) Complete the given series: 4,8, 28, 80, 244, ?

(a) 278 (b) 428 (c) 628 (d) 728 (4)

(B) Choose the Odd pair of words.

(a) China : Beijing (b) Russia : Moscow
(c) Japan : Singapore (d) Spain : Madrid (3)

(C) In a certain code, GLAMOUR is written as IJCNMWP and MISRULE is written as OGUSSNC, then how will TOPICAL be written in that code? (7)

Q 2 (A) Pointing out to a lady, a girl said, "She is the daughter-in-law of the grandmother of my father's only son." How is the lady related to the girl?

(a) Sister-in-law (b) Mother (c) Aunt
(d) Mother-in-law (e) Cousin (4)

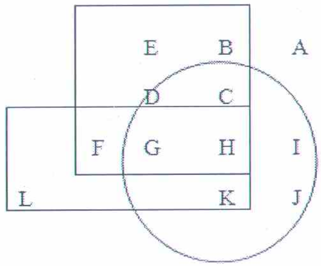
(B) Choose a number set similar to a given number set; 11: 1210

(a) 8: 448 (b) 6: 2160 (c) 7: 1029 (d) 9: 729 (3)

(C) Identify the wrong number in the series: 69, 55, 26, 13, 5 (7)

Section-B

Q3 (A) In the following diagram, the square represents girls, circle tall persons, the triangle is for tennis players and the rectangle stands for the swimmers.



(7)

On the basis of the above diagram, answer the following questions:

(i) Which letter represents tall girls who are swimmers but don't play tennis?

(a) C (b) D (c) G (d) H

(ii) Which letter represents girls who are swimmers, play tennis but are not tall?

(a) B (b) E (c) F (d) None of These

(iii) Which letter represents tall girls who do not play tennis and are not swimmers?

(a) C (b) D (c) E (d) G

(iv) Which letter represents tall persons who are gents and swimmers but do not play tennis?

(a) I (b) J (c) K (d) L

(B) Five persons are standing in a line. One of the two persons at the extreme ends in a professor and the other a business man. An advocate is standing to the right of the student. An author is to the left of the business man. The student is standing between the professor and the advocate. Counting from t he left, the advocate is at which place?

(a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 5^{th} (7)

Q4 (A) If the given interchanges namely: signs + and ÷ numbers 2 and 4 are made in signs and numbers, which one of the following four equations would be correct?

(a) $2 + 4 \div 3 = 3$ (b) $4 + 2 \div 6 = 1.5$

(c) $4 \div 2 + 3 = 4$ (d) $2 + 4 \div 6 = 8$ (7)

(B) If you are facing north-east and move 10m forward, turn left and move 7.5m, then you are

(a) north of your initial position

(b) south of your initial position

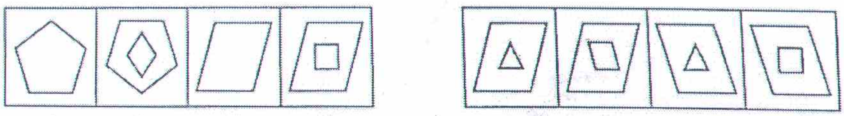
(c) 12m from your initial position

(d) east of your initial position

(e) Both (c) and (d) (7)

Section-C

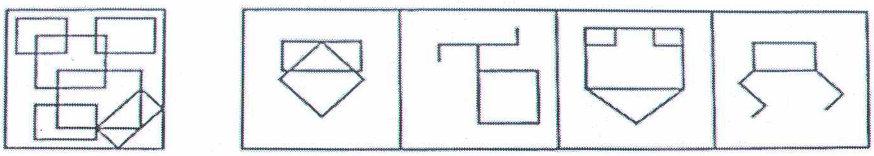
Q5 (A) Following figure consists of four figures marked as A, B, C and D which constitute the problem set followed by four other figures marked as 1, 2, 3 and 4 which constitute the answer set. Set figures A and B are related in a particular manner. Establish a similar relationship between figure C and D by choosing a figure from the answer set.



A B C D 1 2 3 4

(7)

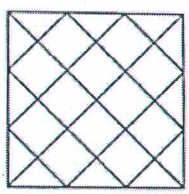
(B) Which figure is embedded in the pattern given in fig. (X)?



(X) (*a*) (*b*) (*c*) (*d*)

(7)

Q6 (A) Find the number of triangles in the given figure after labeling A to Y



(7)

(B) Give synonym of the following:

(i) Candid (ii) Diminish (iii)Feeble (iv) Genuine

Give antonym of the following:

(i) Ancient (ii) Cruel (iii) Expand (7)

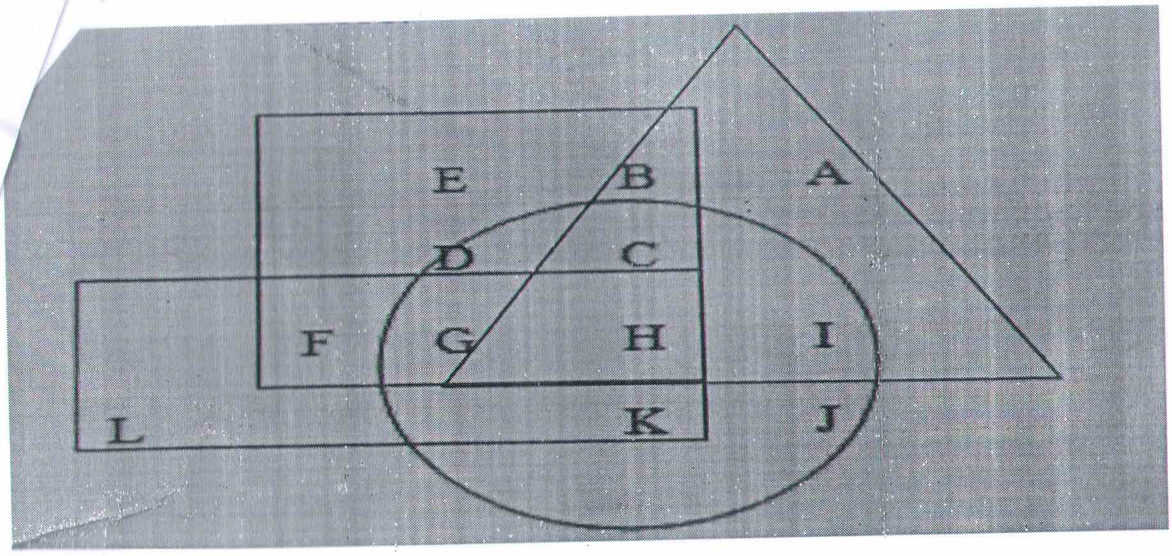
Section-D

Q7 (A) Complete the given figure matrix

Section-B

gram)

6



(For Reappear Candidates Only (2024-25))

Exam Code: 508403 **Paper Code: 9328**

Programme: Master of Science (Mathematics) (FYIP) (Semester: III)

Course Title: Linear Algebra

Course Code: FMAL-3334

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all selecting at least one question from each section. The fifth question can be attempted from any section. Each question carries 16 marks.

Section-A

1.(a) Define a vector space and its subspace and give examples of each.

(b) Prove or disprove that the intersection of two subspaces of a vector space is also a subspace. [8 X 2=16]

2. (a) Determine whether or not the set of vectors (1, 2, 3), (2, 4, 6), (1, 1, 1) in $\mathbb{R}^3$ are linearly independent.
   (b) Prove or disprove that the union of two subspaces is a subspace only if one is contained in the other. [8 X 2=16

Section-B

3. (a) Define basis and dimension of a vector space with an example.

   (b) Justify whether or not that any two bases of a finite-dimensional vector space have the same number of element. [8 X 2=16]

4. (a) Let U and W be subspaces of $\mathbb{R}^4$ given by $U = \{(a, b, c, d): b + c + d = 0\}$ and $W = \{(a,b,c,d) : a + b = 0, c = 2d\}$. Find basis and dimension of $U \cap W$.
   (b) Define quotient space and direct complement of subspace. Give one example for each. [8 X 2=16

Section-C

5. (a) Define a linear transformation. Show that $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (x_2, 0)$ is linear.

   (b) Find a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ whose range space is spanned by (1, 0, 1) and (0, 1, 1). [8 X 2=16

6. (a) State and prove the Rank–Nullity Theorem.

   (b) Find the matrix of $T(x, y) = (3x - 2y, x + 5y)$ relative to the basis $B = \{(1, 3), (3, 4)\}$. [8 X 2=16

Section-D

7. (a) Define an elementary matrix and give an example.

   (b) Prove or disprove that the elementary row operations preserve the rank of a matrix. [8 X 2=16

8. (a) Reduce the following matrix to its normal form and find its rank.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}.$$

(b) Solve the system of equations:

$x + 2y - z = 1,\ 2x + y + 2z = 3,\ x - 4y + 7z = 4.$ [8 X 2=16

Exam Code: 508403 Paper Code: 3197

Programme: Master of Science (Mathematics) (FYIP)

Semester – III

Course Title: Basics of Python Programming

Course Code: FMAM-3334

Time Allowed: 3 Hours Max. Marks: 40

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question may be attempted from any section. Each question carries 8 marks. The use of scientific calculators is permitted (non-programmable). Draw neat, well-labeled diagrams wherever required.

SECTION-A

Q1. (a) Explain the main features of Python that make it popular for scientific computing. (3)

(b) Discuss different data types available in Python with suitable examples. (2)

(c) Write a Python program to find the largest of three numbers using conditional statements. (3)

Q2. (a) What are mutable and immutable objects in Python? Explain with examples. (2)

(b) Differentiate between tuple and list. (3)

(c) Write a short program to interchange keys and values in a dictionary. (3)

SECTION-B

Q3. (a) Explain different types of loops in Python with syntax and example. (2)

(b) Write a program using nested loops to print the following pattern:

1
12
123
1234

(3)

(c) Discuss recursion in Python. Write a recursive function to find the factorial of a number. (3)

Q4. (a) Explain various categories of function arguments in Python with examples. (2)

(b) What is a lambda function? Write a lambda expression to compute the cube of a number. (3)

(c) Using the math module, write code to compute square root, log10, and power of a number. (3)

SECTION-C

Q5. (a) Differentiate between module and package in Python. (3)

(b) Explain the use of dir() and reload() functions. (2)

(c) Write a Python program that creates and imports a user-defined module for calculating area of rectangle and circle. (3)

Q6. (a) What is the purpose of namespaces and scope resolution in Python? (2)

(b) Write a Python code using datetime module to display current date and time in a formatted string. (3)

(c) Describe trigonometric and angular functions available in the math module. (3)

SECTION-D

Q7. (a) Explain file handling modes in Python. (3)

(b) Write a Python program to read a text file and count the number of lines, words, and characters. (3)

(c) What are directories in Python? Write a short code to create and delete a directory. (2)

Q8. (a) Discuss matrix operations in Python using the NumPy library. (3)

(b) Write code to compute the determinant and inverse of a 3×3 matrix using NumPy. (3)

(c) Explain eigenvalues and their importance in linear algebra applications. (2)