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Exam Code: 112416 **Paper Code: 6175**
(30)

Programme: Bachelor of Science (Honours) Mathematics
Semester-VI

Course Title: Complex Analysis

Course Code: BOML-6331

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five question in all selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal marks. (16)

Section-A

1(a) Examine the nature of the function $f(z) = \frac{x^2y^5(x+iy)}{x^4+y^{10}}, z \neq 0, f(0) = 0$ in the region including origin.

(b) Prove that the function $f(z) = u + iv$ is analytic if the first order partial derivatives of u and v not only exists but also continuous functions of x and y and satisfy Cauchy Riemann conditions. (8,8)

2(a) Show that the function $f(z) = |z|$ is not differentiable anywhere.

(b) Show that $u = \frac{1}{2}log(x^2 + y^2)$ is harmonic and find its harmonic conjugate. (8,8)

Section-B

3(a) State and Prove Cauchy Integral formula for derivative of an analytic function.

(b) Evaluate $\int_C \frac{dz}{z(z+\pi i)}$,where C is the circle |z+3i| = 1 (8,8)

4 State and prove Liouville's theorem and hence prove that every polynomial of degree n has exactly n zeroes. (16)

Section-C

5 (a)State and prove Taylor's theorem.

(b) Find Taylor or Laurent expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ *in the regions* $1 < |z| < 2$ *and* $|z| > 1$ (8,8)

6(a) Find the nature of singularities and residues of the function $f(z) = \frac{z}{sinz-tanz}$ *at* $z = 0$

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(b) If a function $f(z)$ is analytic except at finite number of singularities (including at infinity), then prove that the sum of the residues of these singularities is zero. (8,8)

Section-D

7(a) Use Rouche's theorem to show that three out of four zeros of $z^4 + 6z + 3 = 0$ lie in $1 < |z| < 2$

(b) State and prove Argument Principle. (8,8)

8(a) Find the general bilinear transformation which transform the circular disc. $|z| \leq \rho$ on to $|w| \leq \rho'$

(b) Find the Bilinear transformation which maps z = 1, 0, -1 of z plane into w = i, 0,-I of w plane. (8,8)

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Exam Code: 112416 Paper Code: 6176
(30)

Programme: Bachelor of Science (Honours) Mathematics Semester-VI

Course Title: Analytical Skills

Course Code: BOML-6332

Time Allowed: 3 Hours Max Marks: 80

Note: Attempt five question in all selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal marks. (16)

Section -A

Q.1 (i) Find the wrong number in the series : 64,71,80,91,104,119,135,155

(ii) Find the Missing number : 8,7,11,12,14,17,17,22, ____

(iii) In this question there is some relationship between the two terms to the left of :: and the same relationship holds between the two terms to its right . Find out this term

? : ALKLO :: WOULD :TLRIA

(a) BLOCK (b)BARGE (c) CONES (d) DONOR

(iv) Choose that set of numbers from the four alternative sets, that is similar to the given set

Given set : (14,23,32)

(a) (15,23,31) (b) (14,19,24) (c) (13,21,29) (d) (12,21,30)

(4,4,4,4)

Q.2 (i)What was the day of week on 17th june , 1998 ?

(ii) At what time between 5 and 6'o clock are the hands of the clock 3 minutes apart?

(iii) Pointing to a woman , Naman said, "She is the daughter of the only child of my grandmother ". How is the women related to Naman ? (5,6,5)

Section-B

Q.3 (i) Find the common factor of $(127^{127} + 97^{127})$ and $(127^{97} + 97^{97})$

(ii) Seema , Meena and Reena begin to jog around a circular stadium and they complete their revolutions in 54 seconds, 42 seconds and 63 seconds resp. After how much time will they come together at the starting point ?

(iii) If $\frac{b}{a} = 0.25$, then what is the value of $\frac{2a-b}{2a+b} + \frac{2}{9}$? (5,6,5)

Q.4 (i) A Batsman makes a score of 87 runs in the 17th inning and thus increases his average by 3. Find his average after 17th inning .

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(ii) A man can row 40km upstream and 55km downstream in 13 hrs. also, he can row 30Km upstream and 44km downstream in 10hrs. Find the speed of the man in still water and the speed of the current?

(iii) A thief is spotted by a policeman from a distance of 100 metres. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8Km/hr and that of the policeman 10Km/hr , how far the thief will have run before he is overtaken? (5,5,6)

Section-C

Q.5 (i) The radius of a circle is so increased that its circumference increased by 5% . Find the percentage increase in its area.

(ii) If each edge of a cube is increased by 50% , Find the percentage increase in its surface area.

(iii) A Cone, A Hemisphere and A Cylinder stand on equal bases and have the same height. Find the ratio of their Volumes. (6,5,5)

Q.6 (i) At What rate percent per annum of compound interest will Rps.1600 amount to Rps.1852.20 in 3 years. ?

(ii) A,B and C start a Business each investing Rps.20,000. After % months A withdraw Rps.5000 ,B withdraw Rps.4000 and C invest Rps.6000 more . At the end of the year , a total profit of Rps.69900 was recorded . Find the share of each.

(iii) What a Producer allows 36% commission on the retail price of his product , he earns a profit of 8.8% . What would be his profit percent if the commission is reduced by 24%? (5,6,5)

Section-D

Q.7 Study the table given below and answer the question that follow

Total numbers of the employees in the different Departments of an organisation and percentage of the females and the males

Department	Total no. of Employees	Percentage of Females	Percentage of males
IT	840	45	55
Accounts	220	35	65
Production	900	23	77
HR	360	65	35
Marketing	450	44	56
Customer Service	540	40	60

(i) What is the respective ratio of the number of females in the production department to the n umber of females in the marketing department ?

(ii) What is the ratio of the number of females in the HR and Accounts department together to the number of males in the same department together.?

(iii) What is the total number of employees in all the department together?

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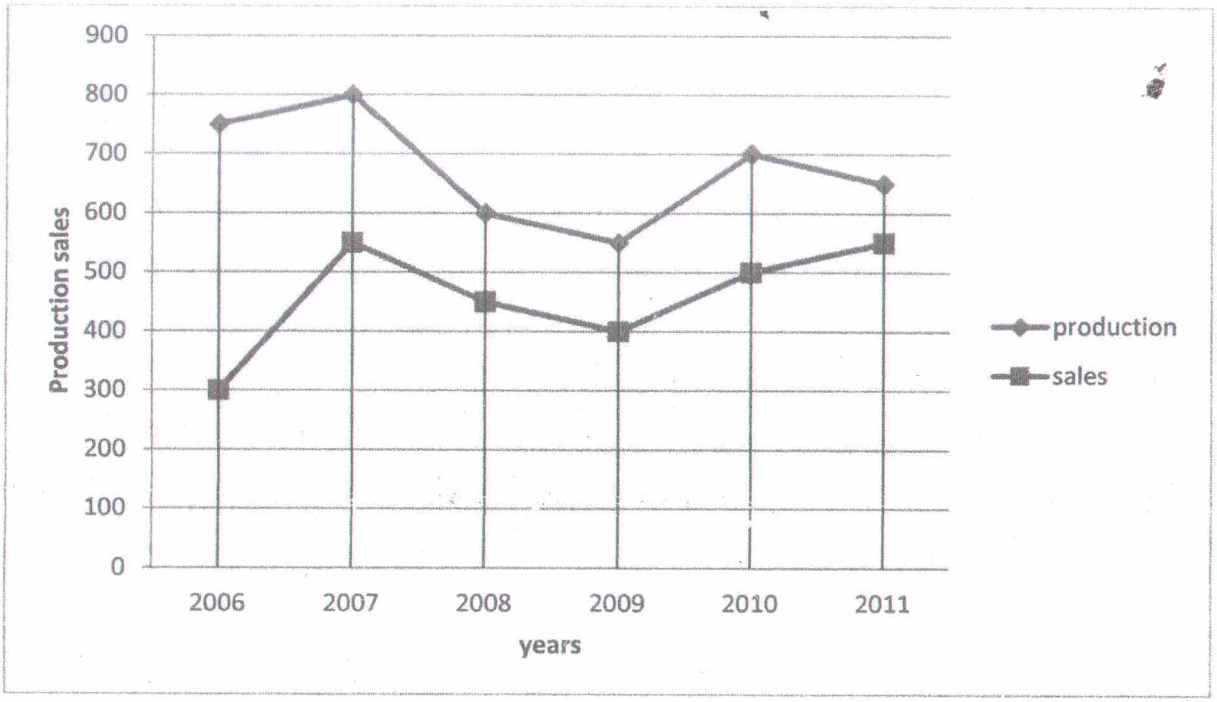
(iv) The total number of the employees in the HR department forms approximately what percent of the total number of the employees in the Accounts department?

(4,4,4,4)

Q.8 Study the following graph and answer the question that follow :

Production (in tonnes) and sales (in tonnes) of company –A from 2006 to 2011



The table given below represent the ratio of the Production (in tonnes) of Company A to the production (in tonnes) of Company B and the ratio of the sales of the company A and Company B

Year	Production (A:B)	Sales (A:B)
2006	5:4	2:3
2007	8:7	11:12
2008	3:4	9:14
2009	11:12	4:5
2010	14:13	10:9
2011	13:14	1:1

(i)What is the approximate percentage increase in the Production of Company A from 2009 to 2010?

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(ii) The sales of Company A in the year 2009 was approximately what percentage of its production in the same year?

(iii) What is the average production of company B (in tonnes) from the year 2006 to the year 2011?

(iv)What is the ratio of the production of company A to the total sales of Company A?

(4,4,4,4)

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Exam Code: 112416 (30)

Paper Code: 6177

Programme: Bachelor of Science (Honours) Mathematics Semester-VI

Course Title: Numerical Analysis

Course Code: BOML-6333

Time Allowed: 3 Hours

Max Marks: 80

Note: Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any Section. Each question carries 16 marks. The students can use non-programmble & non-storage type calculator.

SECTION-A

Q1. (a) Find the root of equation $x^4 - x - 10 = 0$ which lies between 1.5 and 2, correct to three decimal places, using secant method.

(b) If $f = x^3 - 6x$, find the percentage error in f at x=1, if the error in x is 0.04. [8 X 2=16]

Q2. (a) Find the order of convergence of Regula falsi Method.

(b) Find the negative root of the equation $x^3 - 21x + 3500 = 0$ correct to two decimal places between -16 and -15 using Newton- Raphson method.

[8 X 2=16]

SECTION-B

Q3. (a) Solve the equations 2x - 2y + 5z = 13, 2x + 3y + 4z = 20, 3x - y +3z =10 by Gauss Jordan method.

(b) Solve the equations by Gauss Seidel method (use initial approximation as (0, 0, 0))
10x + y + z = 12, 2x + 10y + z = 13, 2x+2y + 10z = 14. [8 X 2=16]

Q4. (a) Using Lagrange interpolation formula, express the function $\frac{x^2+x-3}{x^3-2x^2-x+2}$ as sum of partial fractions. [8 X 2=16]

(b) From the following table estimate the number of students who obtained marks between 70 and 80.

Marks	35-45	45-55	55-65	65-75	75-85
No. of students	18	40	64	50	28

SECTION-C

Q5 (a) Find the first and second order derivatives of $y = f(x)$ at $x = 0$ for the given data.

x	0	1	2	3	4	5
f(x)	4	8	15	7	6	2

(b) Use Stirling's formula to evaluate f (1.22) given [8 X 2=16]

x	1.0	1.1	1.2	1.3	1.4
f(x)	0.841	0.891	0.932	0.963	0.985

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Q6 (a) Evaluate integral $I = \int_0^1 cos2x\,(1-x^2)^{\frac{-1}{2}}\,dx$ by using Gauss Chebyshev three-point formula.

(b) Evaluate $\int_0^1 \int_0^1 e^{x+y}dx\,dy$ using trapezoidal rule with h=k=0.5 [8 X 2=16]

SECTION-D

Q7 (a) Using Picard's method, find an approximate value of y when $x = 0.1$, given that

$\frac{dy}{dx} = x + y^2$ *and* $y = 1$ *when* $x = 0$ $(take\ h = 0.1)$. [8 X 2=16]

(b) Given $\frac{dy}{dx} = x - y^2$ with y(0)=0, y(0.2)=0.02, y(0.4)=0.0795, y(0.6)=0.1762, by Milne's method, compute y(0.8).

Q8. (a) Apply Runge- Kutta method of order 4 to find approximate value of y for x = 0.2, in steps of 0.1, if

$\frac{dy}{dx} = x^2 - y$ *and* $y = 1$ *when* $x = 0$.

(b) Using Taylor series method, find an approximate value of y when $x = 0.25$, given that

$\frac{dy}{dx} = x^2 + y^2$ *and* $y = 1$ *when* $x = 0$ $(take\ h = 0.25)$. [8 X 2=16]

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Exam Code: 112416 **Paper Code: 6178**
(30)

Programme: Bachelor of Science (Honours) Mathematics Semester-VI

Course Title: Special Functions

Course Code: BOML-6334

Time Allowed: 3 Hours **Max Marks: 80**

- **Candidates are required to attempt five question in all, selecting at least one question from each section.**
- **The fifth may be attempted from any section.**
- **All question carry equal marks. (16)**

Section-A

Q1 (a) By applying suitable transformation to the hypergeometirc equation, obtain the identity

$$(1-z)^{-a}F\left[\begin{matrix}a,c-b;\\ c;\end{matrix}-\frac{z}{1-z}\right]=F(a,b;c;z)$$ 8

(b) Show that $\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\frac{d\theta}{(1-x^2\sin^2\theta)^{\frac{1}{2}}}={}_2F_1\left(\frac{1}{2},\frac{1}{2};1;x^2\right),|x|<1$ 8

Q2 (a) Find the solution of $(x-x^2)y''+\left(\frac{3}{2}-2x\right)y'-\frac{y}{4}=0$ about $x=0$. 6

(b) Show that $F(a,b;c,1)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$ 10

Section-B

Q3 (a) Show that $x^nJ_n(x)$ is a solution of $x\frac{d^2y}{dx^2}+(1-2n)\frac{dy}{dx}+xy=0$. 8

(b) Prove that $\frac{d}{dx}\left(x^{-n}J_n(x)\right)=-x^nJ_{n+1}(x)$ 8

Q4 (a) Prove that $J_0^2+2\left(J_1^2+J_2^2+J_3^2+----\right)=1$, where symbol have usual meaning. 8

(b) Show that $J_0(x)=\frac{1}{\pi}\int_0^{\pi}\cos(x\sin\phi)d\phi=\frac{1}{\pi}\int_0^{\pi}\cos(x\cos\phi)d\phi$ 8

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Section-C

Q5 (a) Express $2-3x+4x^2$ in terms of Legendre polynomials. 6

(b) Show that $P_n(x)$ is the coefficient of z^n in the expansion of $(1-2xz+z^2)^{\frac{-1}{2}}$ in ascending power of z. 10

Q6 (a) Prove that $P_n(-x)=(-1)^n P_n(x)$. Deduce that $P_n(-1)=(-1)^n$. 8

(b) Show that $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ 8

Section-D

Q7 (a) Prove that $H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2}$ 10

(b) 1. Express $x^4+2x^3+2x^2-x-3$ in terms of Hermite's polynomials. 6

Q8 (a) Prove that $H_{n+1}(x)=2xH_n(x)-2nH_{n-1}(x)(n\geq 1)$; $H_1(x)=2xH_0(x)$. 8

(b) Prove that $\int_{-\infty}^{\infty} e^{-x^2}H_n(x)H_m(x)dx=2^n n!\sqrt{\pi}\,\delta_{nm}$ 8

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Exam Code: 112416 **Paper Code: 6179**
(30)

Programme: Bachelor of Science (Honours) Mathematics
Semester-VI

Course Title: Differential Geometry

Course Code: BOML-6335

Time Allowed: 3 Hours **Max Marks: 80**

There are total Eight questions of equal marks (16 marks each), two in each of the four Sections (A-D). Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section.

Section A

1. (a) Write a note on three fundamental unit vectors associated with a space curve. Also discuss their relation with three fundamental planes. 8
   (b) Find the equation of tangent line to a curve at a given point in vector and Cartesian form. 8
2. (a) Prove that the tangent at any point of the curve whose equations are $x = 3t, y = 3t^2, z = 2t^3$ makes a constant angle with the line $y = z - x = 0$. 8
   (b) Find the equation of osculating plane in Cartesian form. 8

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Section B

3. (a) The necessary and sufficient condition that a given curve to be a plane curve is torsion is zero at all points. 8

   (b) Show that the curve $\vec{r} = \left(t, \frac{1+t}{t}, \frac{1-t^2}{t}\right)$ lies in a plane

4. (a) Show that necessary and sufficient condition that a curve be a helix is that $\left[\vec{r}'', \vec{r}''', \vec{r}^{iv}\right] = k^5 \frac{d}{ds}\left[\frac{\tau}{k}\right] = 0$. 8

   (b) Prove that the tangent to the locus of the centre of curvature lies in the normal plane of the original curve. 8

Section C

5. (a) Define one parameter family of surfaces and envelopes. Also discuss the working rule to find the envelopes for one parameter family. 8

   (b) Find the fundamental magnitudes and normal to the surface $x = u\cos\emptyset, y = usin\emptyset, z = f(\emptyset)$. 8

6. Find the equation of the tangent plane to the surface $f(x, y, z) = 0$. Using this find the tangent plane to the surface $z = x^2 + y^2$ at the point (1, —1,2). 16

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Section D

7. (a)There exists no distinction between contravariant and co-variant vectors when we restrict ourselves to rectangular Cartesian transformation of coordinates. 8

   (b) A covariant tensor has components $xy, 2y - z^2, xz$ in rectangular Cartesian coordinates. Determine its components in spherical coordinates. 8

8. (a) Find the metric of a Euclidean space referred to spherical coordinates. 8

   (b) State and prove quotient law of tensors. 8