For Reappear Candidates only (2023-24).

Exam Code: 508404 **Paper Code: 9424.**
(20)

Programme: Bachelor of Science (Honours) Mathematics
Semester-IV

Course Title: Group Theory

Course Code: BOML-4333

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting atleast one question from each section and the fifth question may be attempted from any section. Each questions carry 16 marks.

Section-A

1. (a) For n ≥ 1, Show that $\frac{n(n+1)(2n+1)}{6}$ is an integer. 8

   (b) Find integers x, y, z satisfying gcd (198,288,512)= $198x+288y+512z$. 8

2. (a) Prove that each Positive integer n can be uniquely written as $2^k m$ where $k \geq 0$ and m is a positive odd integer. 6

   (b) Let m be a fixed Positive integer. Let R={(a, b): a, b ∈ Z and $a - b$ is divisible by m}.
   Show that R is an equivalence relation on Z. 6

   (c) If gcd (a,b)= lcm[a,b], for non-zero positive integers a,b then.
   Prove that a=b. 4

Section-B

3. (a) Prove that the set $Z_4 = \{0,1,2,3\}$ is not a group under multiplication module 4. 6

   (b) Prove that in a group identify element is unique. 5

   (c) Let $H=\{a + bi : a, b \in R, ab > 0\}$. Prove or disprove that H is a sub group of C under addition. 5

4. (a) Let H be a non empty finite subset of a group G. If H is closed under the operation of G, then H is a subgroup of G. 8

   (b) Let G=GL (2,R) be the group of all 2x2 non-singular matrices over reals under the composition of matrix multiplication. Find the centre of G. 8

Section-C

5. (a) If H and K are finite subgroup of a group G, then $O(HK) = \frac{O(H)O(k)}{O(H \cap K)}$ 8

   (b) Let G be the group $\left\{\begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix} : \text{where } a,b \in R, b \neq o\right\}$ and H $= \left\{\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}, \text{where } x \in R\right\}$. Show that H is a subgroup of G. Is H a normal subgroup of G? Justify your answer. 8

6. (a) Let G be a group such that G/Z (G) is cyclic, where Z (G) is the centre of G. Then Prove that G is a bellian. 8

(b) How many generators are there of the Cyclic group of order. 6

(c) State Logrange's Theorem. 2

Section-D

7. (a) Write down all the elements of S_3. 4

(b) Prove that the order of a Permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. 8

(c) Express the Permutation (1235) (413) as a Product of disjoint cycles. 4

8. (a) Prove that Inverse of even permutation is an even permutation. 8

(b) What cycle is $(a_1 a_2 a_3 ----- a_n)^{-1}$? 2

(c) Show that every element in A_n for $n \geq 3$ can be expresses as a 3-cycle on product of 3-cycles. 6